

[This question paper contains 8 printed pages.]

Your Roll No.

20
May 2025

Sr. No. of Question Paper : 1350

Unique Paper Code : 32351401

Name of the Paper : Partial Differential Equations

Name of the Course : B.Sc. (H) – Mathematics
(CBCS-LOCF)

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Sections are compulsory.
3. Marks of each parts are indicated.

P.T.O.

SECTION - I

Attempt **any two** parts out of the following :

1. (a) Define the Linear and non-linear partial differential equations with examples. Find the first-order partial differential equation satisfied by the family of right circular cones whose axes coincide with z - axes and is given by

$$x^2 + y^2 = (z - c)^2 \tan^2 \alpha. \quad (7.5)$$

- (b) Solve the following initial value problem using method of characteristics:

$$u_t + 2uu_x = v - x, \quad v_t - cv_x = 0,$$

$$\text{with } u(x, 0) = x, \quad v(x, 0) = x. \quad (7.5)$$

- (c) Reduce the following equation into canonical form and then find the general solution:

$$u_x + 2xyu_y = x. \quad (7.5)$$

SECTION - II

Attempt **any one** parts out of the following :

2. (a) Show that the equation of motion of a vibrating string is $u_{tt} = c^2 u_{xx}$, where $c^2 = \frac{T}{\rho}$ and T and ρ are tension per unit length and mass per unit length of string. (6)

- (b) Derive Laplace equation of motion. (6)

Attempt **any two** parts out of the following :

3. (a) Transform the following equation

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0 \text{ to the form } u_{\eta\eta} = 0 \text{ for } y \neq 0. \quad (6)$$

- (b) Determine the general solution of (6)

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

- (c) Find the characteristics, characteristics coordinates and reduce the equation given below to the canonical form; (6)

$$u_{xy} + 2u_{yy} + 9u_x + u = 2.$$

SECTION - III

Attempt any three parts out of the following :

4. (a) Determine the solution of (7)

$$u_{xx} - u_{yy} = 1$$

$$u(x, 0) = \sin(x)$$

$$u_y(x, 0) = x.$$

- (b) Solve the following Goursat problem (7)

$$u_{xy} = h(x, y)$$

$$u(x, 0) = f(x)$$

$$u(0, y) = g(y),$$

where f and g are continuously differentiable, and

$$f(0) = g(0).$$

- (c) Determine the solution of the following vibrating semi-infinite with a fixed end: (7)

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \infty$$

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad 0 \leq x < \infty$$

$$u(0, t) = 0, \quad 0 \leq t < \infty.$$

- (d) Find the solution of the initial boundary value problem (7)

$$u_{tt} - c^2 u_{xx} = 0, \quad t > 0, \quad -\infty < x < \infty,$$

$$u(x, 0) = \log(1 + x^2), \quad u_t(x, 0) = 2, \quad -\infty < x < \infty.$$

SECTION - IV

Attempt **any three** parts out of the following :

5. (a) Show that there exists at most one solution of the heat equation : (7)

$$u_t = k u_{xx}, \quad t > 0, \quad 0 \leq x \leq l,$$

satisfying the initial condition

$$u(x, 0) = f(x), \quad 0 \leq x \leq l,$$

and boundary conditions

$$u(0, t) = 0, \quad u(l, t) = 0, \quad t > 0,$$

- (b) Using the method of separation of variables, discuss the solution of the heat conduction problem : (7)

$$u_t = k u_{xx}, \quad t > 0, \quad 0 \leq x \leq l,$$

$$u(0, t) = 0, \quad u(l, t) = 0, \quad t \geq 0,$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq l.$$

- (c) Find the solution of the following initial boundary value problem : (7)

$$u_{tt} = c^2 u_{xx}, \quad t > 0, \quad 0 \leq x \leq 1,$$

$$u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0,$$

$$u(x, 0) = x(1 - x), \quad u_t(x, 0) = 0, \quad 0 \leq x \leq 1.$$

(d) Solve the problem :

(7)

$$u_{tt} = c^2 u_{xx} + Ax \quad 0 < x < 1, \quad t > 0,$$

$$u(x, 0) = x, \quad u_t(x, 0) = 0, \quad 0 \leq x \leq 1,$$

$$u(0, t) = 0, \quad u(1, t) = 1, \quad t > 0,$$

where A is constant.

This question paper contains 4 printed pages]

23/5/25

Roll No.

S. No. of Question Paper : 1351

Unique Paper Code : 32351402

Name of the Paper : Riemann Integration and Series of Functions

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

All questions are compulsory.

1. (a) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\epsilon > 0$, there exists $\delta > 0$ such that :

$$\text{mesh}(P) < \delta \Rightarrow U(f, P) - L(f, P) < \epsilon,$$

for all partitions P of $[a, b]$.

6

- (b) Show that if f is integrable on $[a, b]$, then $|f|$ is integrable on $[a, b]$, and

$$\left| \int_a^b f \right| \leq \int_a^b |f|.$$

Hence, show that :

$$\left| \int_{-2\pi}^{2\pi} x^2 \sin^8(e^x) dx \right| \leq \frac{16\pi^3}{3}.$$

6

- (c) Show that every continuous function on $[a, b]$ is integrable. Is the converse true ? Justify.

6

P.T.O.

2. (a) Let $f(x) = x^2$, $x \in [0, b]$. Calculate upper and lower Darboux integrals for f on $[0, b]$. Is f integrable on $[0, b]$? 6.5

- (b) If g is a continuous function on $[a, b]$ that is differentiable on (a, b) and if g' is integrable on $[a, b]$, then prove that :

$$\int_a^b g' = g(b) - g(a). \quad 6.5$$

- (c) State Intermediate value theorem for integrals. Suppose f and g are continuous functions on $[a, b]$ such that $\int_a^b f = \int_a^b g$. Prove that there exists $x \in [a, b]$ such that $f(x) = g(x)$. 6.5

3. (a) Show that the function $\beta : \mathbf{R}^+ \times \mathbf{R}^+ \rightarrow \mathbf{R}$ given by :

$$\beta(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$$

is a well-defined function. Find the value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$. 6

- (b) Discuss the convergence or divergence of the following improper integrals :

(i) $\int_1^2 \frac{\sqrt{x}}{\log x} dx$ 3

(ii) $\int_1^\infty \frac{2x^2}{x^4 - 1} dx$. 3

- (c) Let f be a function defined on $[a, b]$ such that for each $x \in (a, b)$, f is bounded and integrable on $[a, x]$ and f has an infinite discontinuity at b . Show that if $\int_a^b |f|$ converges, then so does $\int_a^b f$. 6

4. (a) Show that a sequence $\langle f_n \rangle$ of functions on $A \subseteq \mathbf{R}$ converges uniformly to f on A if and only if $\|f_n - f\|_A \rightarrow 0$. Deduce that the sequence $\langle f_n \rangle$ where $f_n(x) = x/n$ is uniformly convergent on $[0, 1]$. 6.5

- (b) Let $\langle f_n \rangle$ be a sequence of bounded functions on $A \subseteq \mathbf{R}$ that converges uniformly to a function f on A . Show that the function f is bounded on A . 6.5

- (c) Discuss the uniform convergence of the sequence $\langle f_n \rangle$ on $[0, 1]$, where

$$f_n(x) = nxe^{-nx^2} \quad \forall x \in [0, 1]. \quad 6.5$$

5. (a) Show that the series of function $\sum_{n=1}^{\infty} \frac{1}{n^2 x^2} x \neq 0$ converges uniformly for $|x| \leq a$ for some $a > 0$, but is not uniformly convergent on $\mathbf{R} \setminus \{0\}$. 6.5

- (b) State and prove Weierstrass M-test and hence examine the uniform convergence of the series :

$$\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right), \text{ for } x \in [-a, a], \text{ for all } a > 0. \quad 6.5$$

- (c) Prove that the series :

$$\sum_{n=1}^{\infty} \frac{\cos(x^2 + 1)}{n^2}$$

represents a continuous function on $\mathbf{R}$. 6.5

6. (a) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$ (possibly $R = +\infty$). If there exists a real number R_1 , such that $0 < R_1 < R$, then show that the power series $\sum_{n=0}^{\infty} a_n x^n$ is convergent uniformly on $[-R_1, R_1]$. 6

(b) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. Then show that the function f is integrable on $(-R, R)$. Also, show that for $x \in (-R, R)$, we have :

$$\int_0^x f(t) dt = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}. \quad 6$$

(c) Find the exact interval of convergence of the following power series :

$$(i) \quad \sum_{n=1}^{\infty} \frac{x^n}{\ln n} \quad 3$$

$$(ii) \quad \sum_{n=1}^{\infty} \frac{n^n}{n!} x^n. \quad 3$$

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1352

Unique Paper Code : 32351403

Name of the Paper : Ring Theory and Linear Algebra – I

Name of the Course : CBCS B.Sc. (Hons.)
Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each of the question.

1. (a) Define a nilpotent element in a ring. Show that the nilpotent elements of a commutative ring R form a subring of R .

(b) Give an example of a commutative ring which is not an integral domain. Show that a finite integral domain is a field.

(c) In a commutative ring of characteristic 2, prove that the idempotents form a subring. (6,6,6)

P.T.O.

2. (a) Define prime ideal of a commutative ring. Let R be a commutative ring with unity and A be an ideal of R . Then, show that R/A is an integral domain if and only if A is prime ideal of R .

(b) Show that $\frac{\mathbb{R}[x]}{(x^2+1)}$ is a field.

- (c) Define subring of a ring. Let R be the ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Let $A = \{f \in R : f(0) \text{ is an even integer}\}$. Show that A is a subring of R , but not an ideal of R .

(6.5,6.5,6.5)

3. (a) Define a ring homomorphism from a ring R to a ring S . Determine all ring homomorphisms from $\mathbb{Z}_6$ to $\mathbb{Z}_{18}$.

- (b) Let R be a ring with unity 1. Prove that the mapping $\phi: \mathbb{Z} \rightarrow R$ given by $\phi(n) = n \cdot 1$ is a ring homomorphism. Hence, show that if R has characteristic $n > 0$, then R contains a subring isomorphic to $\mathbb{Z}_n$.

- (c) Let $\mathbb{Z}[\sqrt{3}] = \{a + b\sqrt{3} : a, b \in \mathbb{Z}\}$ and

$H = \left\{ \begin{pmatrix} a & 3b \\ b & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}$. Show that $\mathbb{Z}[\sqrt{3}]$ and H are isomorphic as rings.

(6,6,6)

4. (a) (i) Let $\mathcal{C}$ be a collection of subspaces of a vector space V and let W denote the intersection of subspaces in $\mathcal{C}$. Prove that VP is a subspace of V .

(ii) Show that $W = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 + a_2 + a_3 = 0\}$ is a subspace of $\mathbb{R}^3$.

- (b) (i) Let S_1 and S_2 are arbitrary subsets of a vector space V . If $S_1 \subseteq S_2$, then show that $\text{span}(S_1) \subseteq \text{span}(S_2)$.

(ii) Check whether the vectors $(1,0,0)$, $(1,0,1)$ and $(1,1,1)$ generate $\mathbb{R}^3$.

- (c) Define a linearly independent subset of a vector space V . Let S be a linearly independent subset of a vector space V , and let v be a vector in V that is not in S . Prove that $S \cup \{v\}$ is linearly dependent if and only if $v \in \text{span}(S)$.

(6.5,6.5,6.5)

5. (a) Let V and W be vector spaces and $T: V \rightarrow W$ be a linear transformation. Then prove that the null space $N(T)$ and the range space $R(T)$ are subspaces of V and W , respectively.

(b) Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be defined by $T(x_1, x_2) = (x_1 - x_2, x_1, 2x_1 + x_2)$. Let β be the standard ordered basis for $\mathbb{R}^2$ and $\gamma = \{(1,1,0), (0,1,1), (2,2,3)\}$ be an ordered basis for $\mathbb{R}^3$. Compute $[T]_{\beta}^{\gamma}$.

(c) Let $T: P_2(\mathbb{R}) \rightarrow P_3(\mathbb{R})$ be a linear transformation defined by

$$T(a + bx + cx^2) = (a - c) + (a - c)x + (b - a)x^2 + (c - b)x^3.$$

Find nullspace $N(T)$ and range space $R(T)$. Also verify Rank-Nullity theorem. (6,6,6)

6. (a) Let V and W be finite-dimensional vector spaces having ordered bases β and γ respectively, and let $T: V \rightarrow W$ be linear. Then for each $u \in V$, show that

$$[T(u)]_\gamma = [T]_\beta^\gamma [u]_\beta.$$

- (b) Let V , W and Z be vector spaces and let $T: V \rightarrow W$ and $U: W \rightarrow Z$ be linear transformations. Then prove that

(i) If UT is one-to-one, then T is one-to-one.

(ii) If UT is onto, then U is onto.

- (c) Let $A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$ and let $\beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ be an ordered

basis for $\mathbb{R}^2$. Find $[L_A]_\beta$ and also find an invertible matrix Q such that $[L_A]_\beta = Q^{-1}AQ$, where L_A is a left-multiplication transformation.

(6.5, 6.5, 6.5)

SL No of QP : 5534
Unique Paper Code : 2352012401

(70)
May 2025

Name of the Paper : Sequences and Series of Functions

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 hours

Maximum Marks : 90



Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. All questions carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Define uniform convergence for sequence of functions (f_n) defined on $\mathbb{R}$. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$.
(b) Show that the sequence $f_n(x) = \frac{nx}{1+n^2x^2}$ converges uniformly in the interval $[a, \infty)$ where $a > 0$ but it does not converge uniformly on the interval $[0, \infty)$.
(c) Let $(f_n), (g_n)$ be sequences of bounded functions on A that converge uniformly on A to f, g , respectively. Show that (f_ng_n) converges uniformly on A to fg .
2. (a) Let $f_n(x): [0,1] \rightarrow \mathbb{R}$ be defined for $n \geq 2$ by

$$f_n(x) := \begin{cases} n^2x & \text{for } 0 \leq x \leq \frac{1}{n} \\ -n^2\left(x - \frac{2}{n}\right) & \text{for } \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & \text{for } \frac{2}{n} \leq x \leq 1 \end{cases}$$

Evaluate pointwise limit f of f_n and show that (f_n) does not converge uniformly to f on the interval $[0,1]$.

(b) Let (f_n) be a sequence of integrable functions on $[a, b]$ and suppose that (f_n) converges uniformly to f on $[a, b]$. Show that f is integrable on $[a, b]$ and

$$\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n.$$

- (c) Show that the sequence $f_n(x) = n^2x^2e^{-nx}$ converges uniformly on $[a, \infty)$ where $a > 0$ but does not converge uniformly on the interval $[0, \infty)$.

3. (a) State and prove Weierstrass M-test for uniform convergence of series of functions.

Hence, check for uniform convergence of series $\sum_{n=1}^{\infty} \frac{1}{x^2+n^2}$, $x \in \mathbb{R}$.

(b) Discuss the pointwise convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{x^{n+1}+1}$, $x \geq 0$. Also, show that the given series is not uniformly convergent on $[0, 1)$ but is uniformly convergent on $[0, \frac{1}{2})$.

(c) Prove that if $\sum f_n$ converges uniformly to f on a domain $D \subseteq \mathbb{R}$ and each f_n is continuous on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ then f is continuous on D .

4. (a) Let f_n be a real valued function on $[a, b]$ that has derivative f'_n on $[a, b]$ for each $n \in \mathbb{N}$. Suppose that the series $\sum f_n$ converges for at least one point of $[a, b]$ and the series of derivatives $\sum f'_n$ converges uniformly on $[a, b]$. Show that there exists a real valued function f on $[a, b]$ such that $\sum f_n$ converges uniformly to f on $[a, b]$. Also, that f has a derivative on $[a, b]$ and $f' = \sum f'_n$.

(b) State and prove the Cauchy criterion for the uniform convergence of series of

functions $\sum f_n$. Use it to prove the non uniform convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2 x^2}$ for $|x| < 1$.

(c) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{x^{n+1}}$ is convergent for $x > 1$ and divergent for $0 \leq x \leq 1$. Also, show that the series converges uniformly on $[a, \infty)$, $a > 1$.

5. (a) (i) Find the exact interval of convergence for the power series $\sum_{n=1}^{\infty} a_n x^n$ where $a_n = \frac{3^n}{n4^n}$.

(ii) Let $f(x) = |x|$ for $x \in \mathbb{R}$. Is there a power series $\sum_{n=0}^{\infty} a_n x^n$ such that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ for all x ? Justify.

(b) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. If $0 < R_1 < R$, then show that the power series converges uniformly on $[-R_1, R_1]$ to a continuous function.

(c) Show that $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$ for $|x| < 1$. Apply Abel's theorem to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$.

6. (a) Define logarithm function L on $(0, \infty)$. Show that the function L satisfies the following:

(i) $L'(x) = \frac{1}{x}$ for $x > 0$.

(ii) $L(xy) = L(x) + L(y)$ for $x > 0, y > 0$.

(b) Show that $\sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}$ for $|x| < 1$. Evaluate $\sum_{n=1}^{\infty} \frac{n}{3^n}$.

(c) Show that there does not exist a sequence of polynomials converging uniformly on $\mathbb{R}$ to $f(x) = e^x$.

[This question paper contains 8 printed pages.]

Your Roll No.

Sr. No. of Question Paper : 5594

J

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : B.A. / B.Sc. (H)

Semester : IV (DSC)

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is **NOT** allowed.

P.T.O.

1. (a) Let $f(x,y) = \frac{\sin(x^2 + y^2)}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Find

the value of $f(0,0)$ for which $f(x,y)$ is continuous at $(0,0)$.

(b) Compute the slope of the tangent line to the graph

of $f(x,y) = \frac{x^2 + y^2}{xy}$ at $P(1, -1, -2)$ in the direction

parallel to

(i) XZ plane

(ii) YZ plane.

(c) Find $\frac{\partial w}{\partial r}$ where $w = e^{2x-y+3z^2}$ and $x = r + s - t$,

$y = 2r - 3s$, $z = \cos(rst)$.

2. (a) Let $f(x,y,z) = ye^{x+z} + ze^{y-x}$. At the point $P(2, 2, -2)$, find the unit vector pointing in the direction of most rapid increase of $f(x,y,z)$.

(b) Find all the critical points of $f(x,y) = (x-1)(y-1)(x+y-1)$ and classify each as a point of relative maximum, point of relative minimum or a saddle point.

(c) Find the minimum value of the function $f(x,y,z) = x^2 + y^2 + z^2$ subject to $4x^2 + 2y^2 + z^2 = 4$.

3. (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy -plane and above by the graph of $z = 2x + 3y$; $R: 0 \leq x \leq 1, 0 \leq y \leq 2$.

(ii) Evaluate $\int_0^1 \int_x^1 e^{y^2} dy dx$.

(b) (i) Sketch the region of integration and write

an equivalent integral with the order of

integration reversed $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy$.

- (ii) Use a double integral for finding the volume of the solid region bounded above by the paraboloid $z = 6 - 2x^2 - 3y^2$ and below by the plane $z = 0$.

- (c) Use a double integral to find the area bounded by the curve $r = 1 + \sin \theta$.

4. (a) Use cylindrical co-ordinates to compute the integral

$\iiint_D z(x^2 + y^2)^{-1/2} dx dy dz$ where D is the solid bounded above by the plane $z = 2$ and bounded below by the surface $2z = x^2 + y^2$.

- (b) (i) Compute the iterated triple integral

$$\int_0^1 \int_0^y \int_0^{\ln y} e^{z+2x} dz dx dy.$$

- (ii) Evaluate the iterated integral

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\cos \phi} \rho^2 \sin \phi d\rho d\theta d\phi.$$

- (c) Evaluate $\iint_R e^{(2y-x)/(y+2x)} dA$

where R is the trapezoid with vertices $(0,2)$, $(1,0)$, $(4,0)$ and $(0,8)$.

5. (a) A wire has the shape of the curve

$$x = \sqrt{2} \sin t \quad y = \cos t \quad z = \cos t \quad \text{for } 0 \leq t \leq \pi$$

If the wire has density $\delta(x, y, z) = xyz$ at each point (x, y, z) , what is its mass?

- (b) Show that the vector field

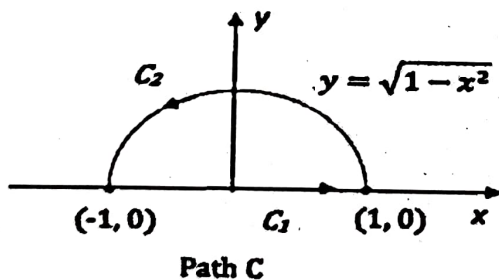
$$F = (e^x \sin y - y)\hat{i} + (e^x \cos y - x - 2)\hat{j}$$

is conservative and hence find a scalar potential function f for F .

(c) Verify Green's theorem for the line integral

$$\oint_C (-y \, dx + x \, dy)$$

where C is the closed path shown in the figure below



6. (a) Evaluate the surface integral

$$\iint_S g \, dS$$

where $g(x, y, z) = xz + 2x^2 - 3xy$ and S is that portion of the plane $2x - 3y + z = 6$ that lies over the unit square $R: 2 \leq x \leq 3, 2 \leq y \leq 3$.

(b) Evaluate

$$\oint_C \left(\frac{1}{2} y^2 \, dx + z \, dy + x \, dz \right)$$

where C is the curve of intersection of the plane $x + z = 1$ and the ellipsoid $x^2 + 2y^2 + z^2 = 1$, oriented counterclockwise.

(c) Use the divergence theorem to evaluate

$$\iiint_S F \cdot N \, dS$$

Where $F = (x^5 + 10x y^2 z^2) \hat{i} + (y^5 + 10y x^2 z^2) \hat{j} + (z^5 + 10z x^2 y^2) \hat{k}$, and S is the closed hemispherical surface $z = \sqrt{1 - x^2 - y^2}$ together with the disc $x^2 + y^2 \leq 1$ in xy -plane and N is the outward unit normal vector field.

This question paper contains 4 printed pages]

03/6/25a

Roll No.

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S. No. of Question Paper : 5673

Unique Paper Code : 2352012403

Name of the Paper : Numerical Analysis

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory. Attempt any two parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. (a) Define order of convergence of a sequence $\{p_n\}$ which converges to a number p . The sequence generated by the formula $x_{n+1} = \frac{x_n^3 + 3x_n a}{3x_n^2 + a}$ converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.
- (b) Perform four iterations of Bisection method to approximate the root of the equation $x^4 - x - 10 = 0$, in the interval $(1, 2)$.
- (c) Explain geometrically the Newton's method to determine a root p of an equation $f(x) = 0$. Hence, using initial approximation $p_0 = 2$, perform two iterations to obtain approximation to the value of $\sqrt[3]{17}$.

P.T.O.

2. (a) Use Newton's method to approximate the root of the equation $\ln(1 + x) - \cos(x) = 0$, in the interval $(0, 1)$. Use error tolerance $= 10^{-4}$, and initial approximation $p_0 = 0.5$.
- (b) Define a fixed point of a function. Let g be a continuous function on the closed interval $[a, b]$, with $g : [a, b] \rightarrow [a, b]$. Furthermore, suppose that g' is continuous on the open interval (a, b) , and there exists a positive constant k such that $|g'(x)| \leq k < 1$, $\forall x \in (a, b)$. Prove that if $g'(p) \neq 0$, then for any $p_0 \in [a, b]$, the sequence $\{p_n\}$, where $p_n = g(p_{n-1})$, converges only linearly to the fixed point p .
- (c) Perform four iterations of Secant method to approximate the root of $x^4 - 7x^2 + 3 = 0$, in the interval $(0, 1)$, using initial approximations $p_0 = 0$ and $p_1 = 1$.
3. (a) For the following system of equations :

$$8x + 5y + 2z = 8,$$

$$6x + 9y + 2z = 11,$$

$$4x + 3y + 8z = 1,$$

starting with the initial approximation as $(0, 0, 0)$, using Gauss Jacobi method, find approximate solution by performing three iterations.

- (b) For the following system of equations :

$$20x + y - 2z = 17,$$

$$3x + 20y - z = -18,$$

$$2x - 3y - 20z = 25,$$

starting with initial approximation as $(0, 0, 0)$, using Gauss Seidel method, find the approximate solution by performing three iterations.

- (c) Find the LU decomposition by taking $u_{ii} = 1$ for the matrix :

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}.$$

4. (a) Construct the Newton form of interpolating polynomial for a function f passing through the points $(0, 1)$, $(1, 3)$ and $(3, 55)$. Hence, evaluate $f(2.5)$.
- (b) Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use Lagrange interpolation to calculate an approximate value for $f(1.04)$ and obtain a bound on the truncation error at $x = 1.04$.
- (c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the data :

x	$f(x)$
1	3
2	7
4	21
8	73

5. (a) Derive the central difference approximation to the second order derivative of a function f at a point $x = x_0$

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}.$$

- (b) Verify that the following difference approximation

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}.$$

for the first order derivative provides the exact value of the derivative, regardless of the values of h , for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

- (c) By using Trapezoidal rule, approximate the value of the integral

$$\int_2^3 (x + \ln x) dx.$$

Further, calculate the absolute error, theoretical error

bound and compare them.

6. (a) Define the degree of precision of a quadrature rule. Further, calculate the degree of precision of Simpson's rule.
- (b) Use Euler's method to determine the approximate solution of the initial value problem $y'(x) = xy^3 - y$, $0 \leq x \leq 2$ such that $y(0) = 2$, by taking the step size as $h = 1/2$.
- (c) Use Modified Euler's method to determine the approximate solution of the initial value problem : $y'(x) = 3x - y/x$, $1 \leq x \leq 2$, such that $y(1) = 2$, by taking the step size as $h = 1/2$.

This question paper contains 7 printed pages]



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S. No. of Question Paper : 5737

Unique Paper Code : 2353012005

Name of the Paper : Mathematical Modeling

Type of the Paper : DSE

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Each question contains three parts.

Attempt *all* questions by selecting *two* parts from each question.

Parts of questions to be attempted together.

All questions carry equal marks.

Use of non-programmable scientific calculators is allowed.

1. (a) Define Mathematical Modeling. Suppose you live on planet of radius R . Place a rope around the planet on its equator. If the length of the rope increased by 10 meters while keeping it in the plane of the equator, how far above the surface of the planet is the lengthened rope ?

P.T.O.

- (b) Define physical and mathematical equations. Convert the following differential equation of decay of an atomic nucleus as in radioactivity, into a dimensionless equation

$$\frac{dc}{dt} = -\lambda c, c(0) = c_0 > 0,$$

where $c(t)$ has the physical units of atoms per unit volume.

- (c) Find the dimensions of the parameters a , D_1 , λ_1 and λ_2 from the following differential equation by assuming it to be dynamically consistent equation :

$$\frac{\partial u}{\partial t} + au \frac{\partial u}{\partial x} = D \frac{\partial^2 u}{\partial x^2} + \lambda_1 \sqrt{u} - \lambda_2 u^2.$$

2. (a) Using mathematical modeling, construct an SIR model. Mention the assumptions and symbols used. Give the properties satisfied by the transition functions $T_1(S, I)$ and $T_2(I)$.
- (b) Consider an epidemic model, where the infected individuals eventually recover and the dynamics of the population are described by the differential equations :

$$\frac{dS}{dt} = -\beta \sqrt{S} \sqrt{I}, \frac{dI}{dt} = \beta \sqrt{S} \sqrt{I} - \gamma \sqrt{I}.$$

Using the parameters values $b = 0.02$ and $g = 0.4$ and assuming initially there is only one infected individual but there are only 500 susceptible individuals within a population :

- (i) Calculate the basic reproduction number r_0 using the given parameters.
 - (ii) How many individuals remain susceptible and never get infected throughout the epidemic ?
 - (iii) What is the maximum number of individuals infected at any point in time during the epidemic ?
- (c) Consider the dieting model $\frac{dM}{dt} = \lambda - \beta M^{\frac{3}{4}}$, where λ, β are constant parameters. Define time scale T^* and show that $T^* = \left(\frac{\lambda}{\beta^4} \right)^{\frac{1}{3}}$. Find the critical point (M^*) for the given dieting model. Is it globally stable ? Draw the graph of $M(t)$ over a period of time if $M(t) > M^*$ and if $M(t) < M^*$.

3. (a) Find the equilibrium solution of $x^3 + (x^2 - 1)x + x = 0$. Discuss the nature and stability of the critical point.

- (b) Explain and analyze the Predator-Prey system by identifying and discussing all isolated critical points.
- (c) Find all critical points of the following systems of differential equations and investigate their type and stability :

$$(i) \quad \frac{dx}{dt} = y^2 - 1, \frac{dy}{dt} = x^3 - y$$

$$(ii) \quad \frac{dx}{dt} = x - y, \frac{dy}{dt} = x + 3y - 4.$$

4. (a) Consider the following system of differential equations,

$$\frac{dx}{dt} = 60x - 3x^2 - 4xy$$

$$\frac{dy}{dt} = 42y - 3y^2 - 2xy.$$

Find the eigenvalues and the corresponding eigenvectors of the coefficient matrix of linear system and hence, determine the type and stability of all non-zero critical points of the system.

- (b) Find the critical point (x^*, y^*) for the system of differential equations :

$$\frac{dx}{dt} = 4x + 6y - 52$$

$$\frac{dy}{dt} = 4x + 3y - 28.$$

Using appropriate transformation, rewrite the system to the following form :

$$\frac{du}{dt} = au + bv$$

$$\frac{dv}{dt} = cu + dv.$$

Analyze the nature and stability of critical points of the original system and the transformed system.

- (c) Consider a damped non-linear spring-mass system. Let m denote the mass of an object attached to a spring and let $x(t)$ be the displacement of the mass at any time t from its equilibrium position. Assume that the mass on the spring is connected to a dashpot, exerting a force of resistance proportional to velocity $y = \dot{x} = \frac{dx}{dt}$ of the mass. Let the force exerted by the spring on the mass be given by $F(x) = m \frac{d^2y}{dt^2} = m\ddot{x}$. The equation of motion of the mass is given by $m\ddot{x} = -c\dot{x} - kx + bx^3$, where $c, m, k, b > 0$. If $m = 1, c = 2, k = 5$ and $b = \frac{5}{4}$, then write the corresponding system of first order differential equations and discuss the stability of all critical points of the system.

5. (a) Using Monte Carlo Simulation, write an algorithm to approximate the area under the curve $f(x) = \sqrt{x}$ over the interval $\frac{1}{2} \leq x \leq \frac{3}{2}$.

- (b) Define random numbers. Explain Middle-Square Method for generating random numbers. Give the drawback if it has any. Use it to generate 15 random numbers using 3043 as the seed number.
- (c) Using Monte Carlo Simulation, write an algorithm to calculate that part of the volume of the sphere $x^2 + y^2 + z^2 \leq 1$ that lies in first octant, $x > 0, y > 0, z > 0$.

6. (a) Use Simplex method to solve the following problem :

$$\text{Max. } Z = 4x + 3y$$

$$\text{s. to } -x + y \leq 6,$$

$$2x + y \leq 20,$$

$$x + y \leq 12,$$

$$x, y \geq 0.$$

- (b) Consider the following Linear Programming Problem :

$$\text{Max. } Z = 6x + 5y$$

$$\text{s. to } x + y \leq 6,$$

$$2x + y \leq 9,$$

$$x, y \geq 0.$$

Using algebraic methods find the possible number of points of intersection in xy -plane. Are all the points feasible ? If not, then how many are feasible and how many are infeasible. List the feasible extreme points along with the value of the objective function.

- (c) Solve the following Linear Programming problem graphically :

$$\text{Max. } Z = 124x + 60y$$

$$\text{s. to } 10x + 3y \leq 16,$$

$$4x + 5y \leq 14,$$

$$x, y \geq 0.$$

Perform sensitivity analysis to find the range of values for the coefficient of y in the objective function for which the current extreme point remains optimal.